YBUS Admittance Matrix Formulation

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This document is a description of how to formulate the YBUS admittance matrix. In general, the diagonal terms Y_{ii} are the self admittance terms and are equal to the sum of the admittances of all devices incident to bus i. The off-diagonal terms Y_{ij} are equal to the negative of the sum of the admittances joining the two buses. Shunt terms only affect the diagonal entries of the Y matrix. For large systems, Y is a sparse matrix and it is structually symmetric.

Transmission Lines

Transmission lines run from bus i to bus j and are indexed by k. More than one transmission line can go between two buses.

$$Y_{ij} = \sum_k \frac{-1}{r_{ijk} + jx_{ijk}} \tag{1}$$

$$Y_{ji} = Y_{ij} \tag{2}$$

$$Y_{ii} = -\sum_{j \neq i} Y_{ij} \tag{3}$$

$$Y_{jj} = -\sum_{i \neq j} Y_{ji} \tag{4}$$

where:
Y_{ij}: the ij_{th} element in the Y matrix.
i: the "from" bus.
j: the "to" bus.
k: the k_{th} transmission line from i to j.
r_{ijk}: the resistance of the k_{th} transmission line from i to j.

x_{ijk}: the reactance of the k_{th} transmission line from i to j.

Transformers

For a tap changing and phase shifting transformer, the off-nominal tap value can in general be considered as a complex number a, where the tap ratio is t and the phase shift is θ. Transformers are defined similarly to transmission lines and exist on a branch between bus i and bus j. Therefore, all transformer elements are subscripted by i and j. For simplicity, we assume that only one transformer line exists between two buses and if a transformer is present between two buses then no transmission lines exist between the two buses. The off-nominal tap ratio between buses i and j is defined as $a_{ij} = t_{ij} * (\cos\theta_{ij} + j * \sin\theta_{ij})$. We can define

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\begin{aligned}
y_{ij}^t &= \frac{-1}{r_{ij} + jx_{ij}} && (5)\\
Y_{ij} &= -\frac{y_{ij}^t}{a_{ij}^*} && (6)\\
Y_{ji} &= -\frac{y_{ij}^t}{a_{ij}} && (7)\\
Y_{ii} &= \frac{y_{ij}^t}{|a_{ij}|^2} && (8)\\
Y_{jj} &= y_{ij}^t && (9)
\end{aligned}
$$

where:
Y_{ij}: the ij_{th} element in the Y matrix.
i: the "from" bus.
j: the "to" bus.
r_{ij}: the resistance of the transformer between i and j.
x_{ij}: the reactance of the transformer between i and j.
t_{ij}: the tap ratio between bus i and bus j.
θ_i: the phase on bus i.
θ_j: the phase on bus j.
$\theta_{ij} = \theta_i - \theta_j$: the phase shift from bus i to bus j.
a_{ij}^*: the conjugate of a_{ij}.

Given the bus admittance matrix Y for the entire system, the transformer model can be introduced by modifying the elements of the Y-matrix derived from the transmission lines as follows:

$$\begin{aligned}
Y_{ij}^{new} &= -\frac{y_{ij}^t}{a_{ij}^*} && (10)\\
Y_{ji}^{new} &= -\frac{y_{ij}^t}{a_{ij}} && (11)\\
Y_{ii}^{new} &= Y_{ii} + \frac{y_{ij}^t}{|a_{ij}|^2} && (12)\\
Y_{jj}^{new} &= Y_{jj} + y_{ij}^t && (13)
\end{aligned}$$

Note that since we are assuming that a branch with a transformer on it does not carry any additional transmission lines, the Y_{ij}^{new} do not have any contributions from Y_{ij}.

For shunts

Shunts only contribute to diagonal elements. The sources of shunts include:

- shunt devices located at buses $(g_i^s + jb_i^s)$;
- transmission line/transformer charging b_{ijk} (distributed half to each end) from end: b_{ik}= $0.5b_{ijk}$; to end: b_{jk}= $0.5b_{ijk}$;
- transmission line/transformer shunt admittance, which is normally a small value: $(g_{ijk}^a + jb_{ijk}^a)$; the shunt admittance contributes the same amount to both ends of the line

Therefore, the general equation for diagonal elements is:

$$Y_{ii}^{tot} = -(\sum_{j\neq i} Y_{ij}^{new}) + g_i^s + jb_i^s + \sum_k (jb_{ki} + g_{ki}^a + jb_{ki}^a) \tag{14}$$

$$Y_{jj}^{tot} = -(\sum_{i\neq j} Y_{ji}^{new}) + g_j^s + jb_j^s + \sum_k (jb_{kj} + g_{kj}^a + jb_{kj}^a) \tag{15}$$

where:
Y_{ij}^{tot}: the ij_{th} element in the Y matrix.
i: the “from” bus.
j: the “to” bus.
k: the k_{th} transmission line/transformer from i to j.
$g_i^s + j * b_i^s$: the shunt at bus i.
b_{ijk}: the line charging of the k_{th} line.
$b_{ik} = 0.5$ *b_{ijk} : the line charging of the k_{th} line assigned to ”from” end i.
$b_{jk} = 0.5$ *b_{ijk} : the line charging of the k_{th} line assigned to ”to” end j.
$g_{ki}^a + b_{ki}^a = g_{ijk}^a + jb_{ijk}^a$: the k_{th} line shunt admittance at ”from” end i.
$g_{kj}^a + b_{kj}^a = g_{ijk}^a + jb_{ijk}^a$: the k_{th} line shunt admittance at ”to” end j.

Thanks!