Look-ahead Dynamic Simulation

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This document is a description of how to formulate the look-ahead dynamic simulation problem using reduced Y-bus formulation for classical model.

Power system dynamic simulation is a critical function for power system transient stability analysis. It computes the system response to a sequence of large disturbance, such as sudden changes in generator or load, or a network short circuit followed by protective branch switching operations.

Basic Equations

The dynamics of a power system can be represented by a set of first-order differential and algebraic equations (DAE):

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}) \\ \mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y}) \end{cases} \tag{1}$$

where the boldface denotes vector, $\mathbf{x}$ represents a vector of state variables (e.g., generator rotor angles and speeds), and $\mathbf{y}$ represents a vector of algebraic variables (e.g., network bus voltage magnitudes and phase angles, or real and imaginary parts of the bus voltage).

When generators are represented using the classical model, the swing equations of generator i in per unit are:

$$\dot{\delta}_{(i)} = \omega_B(\omega_{(i)} - \omega_0) \tag{2}$$

$$\dot{\omega}_{(i)} = \frac{\omega_0}{2H_{(i)}}(P_{m(i)} - P_{e(i)} - D_{(i)}(\omega_{(i)} - \omega_0)) \tag{3}$$

where $\omega_{(i)}$ is the per unit speed for generator i, ω_0 is the per unit synchronous speed, ω_B is the base electrical speed in radians per second, $\delta_{(i)}$ is the angular position of the rotor of generator i in electrical radians with respect to a synchronously rotating reference, $H_{(i)}$ is the inertia constant of generator i normalized by the system base, $P_{m(i)}$ and $P_{e(i)}$ are the mechanical power input and active power at the air gap of generator i, and $D_{(i)}$ is the damping coefficient.

For each generator internal bus i, the injected current in system reference frame can be expressed as

$$I_{(i)} \triangleq I_{re(i)} + jI_{im(i)} = \sum_{k=1}^{n} Y_{(ik)}E_{(k)} \tag{4}$$

where $Y_{(ik)}$ is the (i,k) element in $\mathbf{Y}_{red}$. Let $E_{(k)} = E_{re(i)} + jE_{im(i)}$ and $Y_{(ik)} = G_{(ik)} + jB_{(ik)}$, resolving (4) into real and imaginary part yields

$$I_{re(i)} = \sum_{k=1}^{n}[E_{re(k)}G_{(ik)} - E_{im(k)}B_{(ik)}] \tag{5}$$

$$I_{im(i)} = \sum_{k=1}^{n}[E_{re(k)}B_{(ik)} + E_{im(k)}G_{(ik)}] \tag{6}$$

The transformation from machine $dq0$ reference to system reference frame is

$$\begin{bmatrix} re \\ im \end{bmatrix} = \begin{bmatrix} sin\delta & cos\delta \\ -cos\delta & sin\delta \end{bmatrix} \begin{bmatrix} d \\ q \end{bmatrix}. \tag{7}$$

In classical model, since the d-axis of internal bus voltage $E'_d = 0$, applying the above transformation to (5) and (6), after manipulation, yields

$$I_{re(i)} = \sum_{k=1}^{n}[cos\delta_k E'_{q(k)}G_{(ik)} - sin\delta_k E'_{q(k)}B_{(ik)}] \tag{8}$$

$$I_{im(i)} = \sum_{k=1}^{n}[cos\delta_k E'_{q(k)}B_{(ik)} + sin\delta_k E'_{q(k)}G_{(ik)}] \tag{9}$$

$$I_{q(i)} = cos\delta I_{re(i)} + sin\delta_{im(i)} \tag{10}$$

where $E'_{q(i)}$ is the q-axis of internal bus voltage of generator i, and $I_{q(i)}$ is the injected current. The active power at air gap can be calculated as

$$P_{e(i)} = E'_{q(i)} I_{q(i)} \tag{11}$$

Combining (2), (3) and (11) results in a set of DAE in the same form as (1). The state variables $\mathbf{x}$ are:

$$\mathbf{x} = \left[\delta_{(1)}, \omega_{(1)}, ..., \delta_{(i)}, \omega_{(i)}, ..., \delta_{(n)}, \omega_{(n)}\right]^T \tag{12}$$

and the algebraic variables:

$$\mathbf{u} = \left[I_{d(1)}, I_{q(1)}, ..., I_{d(i)}, I_{q(i)}, ..., I_{d(n)}, I_{q(n)}\right]^T \tag{13}$$

Reduced Y-bus Formulation

Let $\mathbf{Y}$ (with a dimension equal to m by m) denote the nodal admittance matrix of an m-bus system comprised of n generator buses and $m-n$ load buses. After adding machine internal buses and include load impedance into the admittance matrix, resulting in an extended Y-bus – $\mathbf{Y}''$ (with a dimension equal to $m+n$ by $m+n$). The network equation becomes:

$$\begin{bmatrix} \mathbf{I}_E \\ \mathbf{0} \end{bmatrix} = \mathbf{Y}' \begin{bmatrix} \mathbf{E} \\ \mathbf{V} \end{bmatrix}, \tag{14}$$

where $\mathbf{Y}' = \begin{bmatrix} \mathbf{Y}_{nn} & \mathbf{Y}_{nm} \\ \mathbf{Y}_{mn} & \mathbf{Y}_{mm} \end{bmatrix}$, $\mathbf{I}_E$ is the injection currents, $\mathbf{E}$ is the internal generator voltages, and $\mathbf{V}$ is the bus voltages. The network equations is reduced to

$$\mathbf{I}_E = \mathbf{Y}_{red}\mathbf{E}, \tag{15}$$

where $\mathbf{Y}_{red} = \mathbf{Y}_{nn} - \mathbf{Y}_{nm}\mathbf{Y}_{mm}^{-1}\mathbf{Y}_{mn}$.

Woodbury Method

The Woodbury lemma[1] claims that *the inverse of a rank-k correction of a matrix can be computed by doing a rank-k correction to the inverse of the*

[1] M. A. Woodbury, "Inverting modified matrices", Memorandum Rept. 42, Statistical Research Group, Princeton University, Princeton, NJ, 1950, 4pp.

original matrix, where k is significantly smaller than the size of the original matrix.

Given $\mathbf{X} = \mathbf{Y}_{mm}^{-1}\mathbf{Y}_{mn}$ and a switching action causing an incremental change $\Delta\mathbf{Y}_{mm}$ to the system admittance matrix $\mathbf{Y}_{mm}$, the following formula is applied to compute $(\mathbf{Y}_{mm} + \Delta\mathbf{Y}_{mm})^{-1}\mathbf{Y}_{mn}$, using

$$(\mathbf{Y}_{mm} + \mathbf{UBV})^{-1}\mathbf{Y}_{mn} = \mathbf{Y}_{mm}^{-1}\mathbf{Y}_{mn} - \mathbf{Y}_{mm}^{-1}\mathbf{U}(\mathbf{B}^{-1} + \mathbf{V}\mathbf{Y}_{mm}^{-1}\mathbf{U})^{-1}\mathbf{V}\mathbf{Y}_{mm}^{-1}\mathbf{Y}_{mn} \tag{16}$$

where $\Delta\mathbf{Y}_{mm} = \mathbf{UBV}$, and $\mathbf{U}$, $\mathbf{B}$, and $\mathbf{V}$ can be formulated based on the incremental change (such as line tripping). Since $\mathbf{Y}_{mm}^{-1}\mathbf{Y}_{mn}$ is already computed from the original matrix, and $\mathbf{U}$, $\mathbf{B}$, and $\mathbf{V}$ are low dimensional highly sparse matrices, the computational cost involved in (16) is much lower than inverting $\mathbf{Y}_{mm}$ itself.

Numerical Integration

The modified Euler Method is used for the numerical integration in the dynamic simulation process.